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EXAMPLES ON FIBERS OF RATIONAL MAPS

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ABSTRACT

A rational map $\phi: \mathbb{P}_k^m \dashrightarrow \mathbb{P}_k^n$ is defined by homogeneous polynomials of a common degree d. In [4, 9], the authors have established some bounds in terms of d for the number of $(m-1)$-dimensional fibers of ϕ. An interesting question is whether the number of points in $\mathbb{P}_k^3$ with one-dimensional fibers under a rational map $\phi: \mathbb{P}_k^2 \dashrightarrow \mathbb{P}_k^3$ can be arbitrarily large. In this paper, we show that the number of one-dimensional fibers can reach the bound $2d-2$, thereby proving the sharpness of previously known estimates.

Keywords: fibers of rational maps, parameterizations.

1. INTRODUCTION

Let k be a field and $\phi: \mathbb{P}_k^m \dashrightarrow \mathbb{P}_k^n$ be a rational map. Such a map ϕ is defined by homogeneous polynomials $f_0, \dots, f_n$ of the same degree d in the standard graded polynomial ring $R = k[X_0, \dots, X_m]$, satisfying $\gcd(f_0, \dots, f_n) = 1$. The ideal I of R generated by these polynomials is called the *base ideal of* ϕ. The subscheme $\mathcal{B} := \mathrm{Proj}(R/I) \subset \mathbb{P}_k^m$ is called the *base locus of* ϕ. Let $B = k[T_0, \dots, T_n]$ be the homogeneous coordinate ring of $\mathbb{P}_k^n$. The map ϕ corresponds to the k-algebra homomorphism $\varphi: B \to R$, which sends each T_i to f_i. Then the kernel of this homomorphism defines the closed image $\mathcal{S}$ of ϕ. In other words, after degree renormalization, $k[f_0, \dots, f_n] \simeq B/\mathrm{Ker}(\varphi)$ is the homogeneous coordinate ring of $\mathcal{S}$. The minimal set of generators of $\mathrm{Ker}(\varphi)$ is called its *implicit equations* and the *implicitization problem* is to find these implicit equations.

The implicitization problem for curves and surfaces has increasingly attracted the interest of commutative algebraists and algebraic geometers due to its applications in Computer Aided Geometric Design as explained by Cox [6].

We blow up the base locus of ϕ and obtain the following commutative diagram

$$\begin{array}{ccc} \Gamma & \hookrightarrow & \mathbb{P}_k^m \times \mathbb{P}_k^n \\ \downarrow \pi_1 & & \downarrow \pi_2 \\ \mathbb{P}_k^m & \overset{\phi}{\dashrightarrow} & \mathbb{P}_k^n \end{array}$$

The variety Γ is the blow-up of $\mathbb{P}_k^m$ at $\mathcal{B}$ and it is also the Zariski closure of the graph of ϕ in $\mathbb{P}_k^m \times \mathbb{P}_k^n$. Moreover, Γ is the geometric version of the Rees algebra $\mathcal{R}_\mathrm{I}$ of I, i.e. $\mathrm{Proj}(\mathcal{R}_\mathrm{I}) = \Gamma$. As $\mathcal{R}_\mathrm{I}$ is the graded domain defining Γ, the projection $\pi_2(\Gamma) = \mathcal{S}$ is defined by the graded domain $\mathcal{R}_\mathrm{I} \cap k[T_0, \ldots, T_n]$ and we can thus obtain the implicit equations of $\mathcal{S}$ from the defining equations of $\mathcal{R}_\mathrm{I}$.

In geometric modeling, it is of vital importance to have a detailed knowledge of the geometry of the objects and of the parametric representations one is working with. The question of how many times the same point is being painted (i.e. corresponds to distinct values of parameter) depends not only on the variety itself, but also on the parameterization. It is of interest to determine the singularities of the parameterizations, in particular their fibers. More precisely, we set

$$\pi := \pi_{2|\Gamma} : \Gamma \to \mathbb{P}_k^n.$$

For every closed point $y \in \mathbb{P}_k^n$, we will denote by $k(y)$ its residue field. If k is assumed to be algebraically closed, then $k(y) \simeq k$. The fiber of π at $y \in \mathbb{P}_k^n$ is the subscheme

$$\pi^{-1}(y) = \mathrm{Proj}\left(\mathcal{R}_\mathrm{I} \bigotimes_B k(y)\right) \subset \mathbb{P}_{k(y)}^m \simeq \mathbb{P}_k^m.$$

Suppose that $m \geq 2$ and ϕ is generically finite onto its image. Then the set

$$\mathcal{Y}_{m-1} = \{y \in \mathbb{P}_k^n \mid \dim \pi^{-1}(y) = m - 1\}$$

consists of only a finite number of points in $\mathbb{P}_k^n$. For each $y \in \mathcal{Y}_{m-1}$, $\pi^{-1}(y)$ is a $(m-1)$-dimensional subcheme of $\mathbb{P}_k^m$ and thus the unmixed component of maximal dimension is defined by a homogeneous polynomial $h_y \in R$. In recent papers [4, 9], the authors have established some bounds for $\sum_{y \in \mathcal{Y}_{m-1}} \deg(h_y)$ in terms of the degree d. In this paper, we construct explicit examples of rational maps $\phi: \mathbb{P}_k^2 \dashrightarrow \mathbb{P}_k^3$ whose fibers attain the maximal number of one-dimensional components permitted by known bounds. Our construction not only provides concrete cases where the upper bound $2d - 2$ is achieved, but also establishes the sharpness of previous estimates in the literature.

2. FIBERS OF A RATIONAL MAP $\boldsymbol{\phi}: \mathbb{P}_{\boldsymbol{k}}^{\boldsymbol{m}} \dashrightarrow \mathbb{P}_{\boldsymbol{k}}^{\boldsymbol{n}}$

For simplicity, we summarize the following data.

Data 2.1. Let k be an algebraically closed field and let $n \geq m \geq 2$ and $d \geq 1$. Let I be minimally generated by homogeneous polynomials $\mathbf{f} := f_0, \dots, f_n$, of degree d, in $\mathrm{R} := k[\mathbf{X}] = k[X_0, \dots, X_m]$ satisfying $\gcd(f_0, \dots, f_n) = 1$. Suppose that $\mathbf{f}$ define a rational map $\phi: \mathbb{P}^m \dashrightarrow \mathbb{P}^n$ that is generically finite onto its image.

In [4], the authors have generalized a result in [1] which gives the structure of the unmixed part of a $(m-1)$-dimensional fiber of π. Recall that the saturation of an ideal J of R is defined by $J^{\mathrm{sat}} := J :_R (\mathbf{X})^\infty$ and the initial degree of a graded R-module M by

$$\mathrm{indeg}(M) := \inf\{n \in \mathbb{Z}: M_n \neq 0\}$$

with the convention that $\sup\emptyset = +\infty$.

Lemma 2.2. [4] *Assume Data 2.1 holds. Let $y = (p_0 : \cdots : p_n) \in \mathcal{Y}_{m-1}$ satisfying $p_i = 1$. Then,*

$$h_y = \gcd(f_0 - p_0 f_i, \dots, f_n - p_n f_i) \text{ and } I = (f_i) + h_y(g_0, \dots, g_{i-1}, g_{i+1}, \dots, g_n),$$

where $f_j - p_j f_i = h_y g_j$ for all $j \neq i$. Moreover, $I^{\mathrm{sat}} \subset (f_i, h_y)$.

The following theorem is a generalization of [9, Proposition 1].

Theorem 2.3. *Assume Data 2.1 holds. If there exists an integer s such that $\nu = \mathrm{indeg}((I^s)^{\mathrm{sat}}) < sd$, then $\sum_{y \in \mathcal{Y}_{m-1}} \deg(h_y) \leq \nu < sd$.* □

Proof. The proof of this result goes along the same lines as in the proof of Proposition 1 in [9], using Lemma 2.2. □

In particular, Theorem 2.3 shows that

$$\sum_{y \in \mathcal{Y}_{m-1}} \deg(h_y) < d,$$

whenever $\mathrm{indeg}(I^{\mathrm{sat}}) < d$. The delicate case is when the ideal I satisfies $\mathrm{indeg}(I^{\mathrm{sat}}) = d$. M. Chardin, S.D. Cutkosky and the second author have proved the following for the parameterizations of surfaces $\phi: \mathbb{P}_k^2 \dashrightarrow \mathbb{P}_k^3$.

Theorem 2.4. [4, 9] *Assume Data 2.1 holds. Assume further that $m = n - 1 = 2$ and $\mathrm{indeg}(I^{\mathrm{sat}}) = d$.*

(i) *If $\mathcal{B} = \mathrm{Proj}(R/I)$ is locally a complete intersection, then*

$$\sum_{y \in \mathcal{Y}_1} \deg(h_y) \leq \begin{cases} 4 & \text{if } d = 3, \\ \left\lfloor \dfrac{d}{2} \right\rfloor d - 1 & \text{if } d \geq 4. \end{cases}$$

(ii) *If the characteristic of k does not divide d and $[k(\mathbf{f}): k(\mathbf{X})]$ is separable, then*

$$\sum_{y\in\mathcal{Y}_1} \deg\left(h_y\right) \leq 3(d-1) - \operatorname{indeg}\big(\operatorname{Syz}(I)\big) < 3(d-1).$$

Recall that if $\mathbf{f} := f_0, \dots, f_n$ are polynomials in $R = k[X_0, \dots, X_m]$, then the Jacobian matrix of $\mathbf{f}$ is defined by

$$J(\mathbf{f}) = \begin{pmatrix} \frac{\partial f_0}{\partial X_0} & \cdots & \frac{\partial f_0}{\partial X_m} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial X_0} & \cdots & \frac{\partial f_n}{\partial X_m} \end{pmatrix}.$$

Let $I_s\big(J(\mathbf{f})\big)$ denote the ideal of R generated by the s-minors of $J(\mathbf{f})$. M. Chardin, S.D. Cutkosky and the second author have generalized the above theorem.

Theorem 2.5. [4] *Assume Data 2.1 holds. Assume further that $I_3\big(J(\mathbf{f})\big) \neq 0$. Let F be the greatest common divisor of generators of $I_3\big(J(\mathbf{f})\big)$. Then*

$$\sum_{y\in\mathcal{Y}_{m-1}} \deg\left(h_y\right) \leq \sum_{y\in\mathcal{Y}_{m-1}} \sum_{i=1}^{r_y} (2e_i - 1)\deg(h_i) \leq \deg(F) \leq 3(d-1),$$

where $h_y = h_1^{e_1} \cdots h_{r_y}^{e_{r_y}}$ is an irreducible factorization of h_y in R.

If the field k is of characteristic zero, then the assumptions $I_3\big(J(\mathbf{f})\big) \neq 0$ and the separability of $[k(\mathbf{f}): k(\mathbf{X})]$ are always satisfied, due to the hypothesis that ϕ is generically finite onto its image.

Remark 2.6.

(i) The inequality

$$\sum_{y\in\mathcal{Y}_{m-1}} \deg\left(h_y\right) \leq \sum_{y\in\mathcal{Y}_{m-1}} \sum_{i=1}^{r_y} (2e_i - 1)\deg(h_i)$$

becomes an equality if and only if the defining equation of the unmixed component of the fiber $\pi^{-1}(y)$ has no multiple factors, for every $y \in \mathcal{Y}_{m-1}$.

(ii) The bound

$$\sum_{y\in\mathcal{Y}_{m-1}} \sum_{i=1}^{r_y} (2e_i - 1)\deg(h_i) \leq \deg(F)$$

is optimal as the following example shows.

In the case of parameterization $\phi: \mathbb{P}_k^2 \dashrightarrow \mathbb{P}_k^3$ of algebraic rational surfaces. Such a map ϕ is defined by four homogeneous polynomials $f_0, \dots, f_3$, not all zero, of the same degree d, in the standard graded polynomial ring $R = k[x, y, z]$. Our goal is to establish a bound for the cardinality of the set of points in $\mathbb{P}_k^3$ with a one-dimensional fiber, that is, the cardinality of the set

$$\mathcal{Y}_1 = \{y \in \mathbb{P}_k^3 \mid \dim \pi^{-1}(y) = 1\}.$$

The following corollary is a direct consequence of Theorem 2.4.

Corollary 2.7. *Assume Data 2.1 holds. Assume further that* $m = n - 1 = 2$ *and* $\operatorname{indeg}(I^{\mathrm{sat}}) = d$.

(i) *If* $\mathcal{B} = \operatorname{Proj}(R/I)$ *is locally a complete intersection, then*

$$\#\mathcal{Y}_1 \le \begin{cases} 4 & \text{if } d = 3, \\ \left\lfloor \dfrac{d}{2} \right\rfloor d - 1 & \text{if } d \ge 4. \end{cases}$$

(ii) *If the characteristic of* k *does not divide* d *and* $[k(\mathbf{f}) : k(x, y, z)]$ *is separable, then*

$$\#\mathcal{Y}_1 \le 3(d-1) - \operatorname{indeg}\big(\operatorname{Syz}(I)\big)\} < 3(d-1).$$

Example 2.8. [9, Example 10] Let $d \ge 4$ be an integer. Consider the parameterization given by $\mathbf{f} = f_0, \dots, f_3$, with

$$f_0 = X_0^{d-3} X_1 (X_0^2 - X_1^2), \qquad f_2 = X_0^{d-3} X_2 (X_1^2 - X_2^2),$$
$$f_1 = X_0^{d-3} X_2 (X_0^2 - X_1^2), \qquad f_3 = X_1^{d-3} X_2 (X_1^2 - X_2^2).$$

By using Macaulay2 [8], we get the greatest common divisor of generators of $I_3\big(J(\mathbf{f})\big)$ to be

$$F = X_0^{2d-7} X_2 (X_0^2 - X_1^2)(X_1^2 - X_2^2).$$

It is known from [9, Example 10] that

$$\sum_{y \in \mathcal{Y}_1} \deg\big(h_y\big) = d + 2$$

and

$$\sum_{y \in \mathcal{Y}_1} \sum_{i=1}^{r_y} (2e_i - 1) \deg(h_i) = 2(d-1) = \deg(F) < 3(d-1).$$

Furthermore, if $d = 4$, then

$$\sum_{y \in \mathcal{Y}_1} \deg\big(h_y\big) = \sum_{y \in \mathcal{Y}_1} \sum_{i=1}^{r_y} (2e_i - 1) \deg(h_i) = \deg(F).$$

3. MAIN RESULTS

An interesting question is: could the number of points in $\mathbb{P}^3_k$ having a fiber of dimension one of a rational map $\phi: \mathbb{P}^2_k \dashrightarrow \mathbb{P}^3_k$ be arbitrarily big? We now construct parameterizations that exhibit a large number of one-dimensional fibers.

Example 3.1. Let k be a field of characteristic zero and $d \geq 3$ be an integer. Let $a_1, \ldots, a_{d-2}, b_1, \ldots, b_{d-2} \in k^* = k\setminus\{0\}$ such that $a_i \neq a_j$ and $b_i \neq b_j$ for all $i \neq j$. Set

$$f = \prod_{i=1}^{d-2}(x - a_i z) \quad \text{and} \quad g = \prod_{i=1}^{d-2}(y - b_i z)$$

two homogeneous polynomials of degree $d-2$. Consider the matrix

$$M = \begin{pmatrix} -z & 0 & g \\ 0 & -z & -f \\ y & 0 & 0 \\ 0 & x & 0 \end{pmatrix}$$

And let f_j be $(-1)^{j+1}$ times the minor obtained from M by leaving out the $(j+1)$-th row, for all $j = 0, \ldots, 3$. Let $I = (f_0, f_1, f_2, f_3)$ be the ideal of $R = k[x, y, z]$. Then, I is a saturated ideal of codimension two. By the Hilbert-Burch theorem, I admits a minimal free resolution of the form

$$0 \longrightarrow R(-d-1)^2 \oplus R(-2d+2) \xrightarrow{M} R(-d)^4 \longrightarrow R \longrightarrow R/I \longrightarrow 0$$

Note that $f_0 = xyf$, $f_1 = xyg$, $f_2 = xzf$ and $f_3 = yzg$. It follows that $\mathbf{f} := f_0, \ldots, f_3$ are k-linearly independent. Since $\gcd(f, g) = 1$, one has $\gcd(f_0, \ldots, f_n) = 1$. Thus, $\mathcal{B} = \operatorname{Proj}(R/I)$ is a zero-dimensional subscheme of $\mathbb{P}^2_k$. Using the vanishing theorems for the sheaf cohomology, one obtain

$$\deg(\mathcal{B}) = \dim_k (R/I)_\mu$$

for $\mu \gg 0$. Since $\dim_k R_\mu = \binom{\mu+2}{2}$, the above resolution for R/I shows that

$$\deg(\mathcal{B}) = \frac{1}{2}(d^2 + (d-2)^2 + 2) = d^2 - 2d + 3.$$

This formula has been proven in [5]. Moreover, it is straightforward to verify that

$$\mathcal{B} = \{(1,0,0), (0,1,0), (0,0,1), (a_i, 0, 1), (0, b_i, 1), (a_i, b_i, 1) \mid i = 1, \ldots, d-2\}.$$

Therefore, $d_{\mathfrak{p}} = 1$ for all $\mathfrak{p} \in \mathcal{B}$. It follows that $\mathcal{B}$ is locally a complete intersection.

Consider the parameterization $\phi: \mathbb{P}^2_k \dashrightarrow \mathbb{P}^3_k$ defined by $\mathbf{f} = f_0, f_1, f_2, f_3$ and let S be the Zariski closure of its image.

Let L_1, L_2, L_3 be a R-basis of the syzygy module of I. Write

$$\begin{aligned} L_1 &= T_2 y - T_0 z \\ L_2 &= T_3 x - T_1 z \\ L_3 &= T_0 g - T_1 f \end{aligned}$$

and $N = \begin{pmatrix} 0 & T_2 & -T_0 \\ T_3 & 0 & -T_1 \end{pmatrix}$. We see that ϕ has a rational inverse ψ given by the (signed) maximal minor of N, that is

$$\begin{aligned} \psi: \quad & S \dashrightarrow \mathbb{P}_k^2 \\ (T_0 : \cdots : T_3) \quad & \mapsto (T_1T_2 : T_0T_3 : T_2T_3), \end{aligned}$$

see, for example, [3], thus $\deg(\phi) = 1$. By the degree formula [5], the image S of ϕ is a surface in $\mathbb{P}_k^3$ of degree

$$\deg(S) = d^2 - \sum_{x \in \mathcal{B}} e_x(I) = d^2 - \deg(\mathcal{B}) = 2d - 3,$$

where $e_x(I)$ is the Hilbert-Samuel multiplicity, as defined in [2]. The equality holds since I is locally a compete intersection. Consequently, the set of one-dimensional fibers is

$$\mathcal{Y}_1 = \{\mathfrak{p}, \mathfrak{q}, \mathfrak{p}_i, \mathfrak{q}_i \mid i = 1, \dots, d-2\},$$

where

$$\begin{aligned} &\mathfrak{p} = (0:0:0:1) && h_{\mathfrak{p}} = x, \\ &\mathfrak{q} = (0:0:1:0) && h_{\mathfrak{q}} = y, \\ &\mathfrak{p}_i = (0:a_i:0:1) && h_{\mathfrak{p}_i} = x - a_i z \quad \forall i = 1, \dots, d-2, \\ &\mathfrak{q}_i = (b_i:0:1:0) && h_{\mathfrak{q}_i} = y - b_i z \quad \forall i = 1, \dots, d-2. \end{aligned}$$

Therefore, $\#\mathcal{Y}_1 = 2(d-1)$. It follows that $\operatorname{indeg}((I^2)^{\text{sat}}) \geq 2(d-1)$.

The following theorem shows that the inequality in Theorem 2.3 is optimal.

Theorem 3.2. *It holds that $xyfg \in (I^2)^{\text{sat}}$. Therefore,* $\operatorname{indeg}((I^2)^{\text{sat}}) = 2(d-1)$.

Proof. Set $\mathfrak{m} = (x, y, z)$. It suffices to show that $\mathfrak{m}^{d-1}xyfg \subset I^2$.

Since $x^2y^2fg, x^2yzfg, xy^2zfg, xyz^2fg \in I^2$, the claim will be completed by showing that $x^d yfg, xy^d fg \in I^2$.

Let us write

$$x^d yfg = Af_0^2 + Bf_0f_2 + Cf_1f_2 = Ax^2y^2f^2 + Bx^2yzf^2 + Cx^2yzfg,$$

which deduces

$$x^{d-2}g = Ayf + Bzf + Czg = (Ay + Bz)f + Czg \Rightarrow \begin{cases} Ay + Bz = g \\ Cz = x^{d-2} - f. \end{cases}$$

Since

$$Ay + Bz = g = \prod_{i=1}^{d-2}(y - b_i z) = \left(\prod_{i=1}^{d-3}(y - b_i z)\right)y - \left(b_{d-2}\prod_{i=1}^{d-3}(y - b_i z)\right)z.$$

We choose $A = \prod_{i=1}^{d-3}(y - b_i z)$ and $B = -b_{d-2}\prod_{i=1}^{d-3}(y - b_i z)$. Since

$$x^{d-2} - f = x^{d-2} - \prod_{i=1}^{d-2}(x - a_i z) = (\sigma_1 x^{d-3} - \sigma_2 x^{d-4} z + \cdots + (-1)^{d-1}\sigma_{d-2} z^{d-3})z,$$

where $\sigma_1, \ldots, \sigma_{d-2}$ are the elementary symmetric polynomials in $d-2$ variables $a_1, \ldots, a_{d-2}$. Thus, we choose $C = \sigma_1 x^{d-3} - \sigma_2 x^{d-4} z + \cdots + (-1)^{d-1}\sigma_{d-2} z^{d-3}$. This shows that $x^d y f g \in (f_0^2, f_0 f_2, f_1 f_2) \subset I^2$.

Similarly, we can prove that $xy^d f g \in (f_1^2, f_0 f_3, f_1 f_3) \subset I^2$. Hence, we conclude that $\operatorname{indeg}((I^2)^{\text{sat}}) = 2(d-1)$ which shows that the bound given in Theorem 2.3 is sharp. This completes the proof. □

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NHỮNG VÍ DỤ VỀ ẢNH NGƯỢC CỦA ÁNH XẠ HỮU TỈ

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TÓM TẮT

Một ánh xạ hữu tỉ ϕ: $P^m \cdots > P^n$ được định nghĩa bởi $n+1$ đa thức thuần nhất có chung bậc d. Trong hai bài báo [4, 9], các tác giả đã thiết lập một vài chặn theo d số của các ảnh ngược chiều $m-1$ của ϕ. Một câu hỏi thú vị đặt ra là: liệu có tồn tại ánh xạ hữu tỉ ϕ: $P^2 \cdots > P^3$ sao cho nó có số lượng lớn các điểm trong P^3 có ảnh ngược 1-chiều hay không. Trong bài báo này, chúng tôi chứng minh rằng số ảnh ngược một chiều có thể đạt đến cận $2d-2$, qua đó khẳng định tính sắc sảo (chặt) của các ước lượng đã biết trước đó.

Từ khoá: Ảnh ngược của ánh xạ hữu tỉ, tham số hoá.

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